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EVALUATION OF COMBAT

by

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EVALUATION OF COMBAT

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ABSTRACT

Assessment of effects of changes in weapons systems or battle tactics is difficult because of the variations in battles and the resulting instability of measures of combat effectiveness. Even in the relatively stable conditions of designed experimentation, traditional measures may fail to reflect important battle events and dynamics, and sample sizes are driven high in an effort to overcome large variances. This variation in results makes the design, conduct and evaluation of combat experimentation a challenging endeavor, indeed. We develop and examine a measure of combat effectiveness, based on Lanchester models, which we call the "battle trace." The battle trace is a measure of ongoing battle results, measured as a function of time into the battle. We discuss how such measures can be used to compare effects of factor levels in designed comparisons, and we describe an application to evaluation of human factors in combat simulations.

1 INTRODUCTION

Measuring and analyzing the effectiveness of combat, from one side's point of view, is a very challenging task. Selecting appropriate measures of effectiveness is a key element of this process. A selected measure is appropriate, when quantified in a particular context, if it illuminates some feature of the subject of the analysis in an insightful way. Such measures give decision makers reasonable criteria with which to distinguish the differences between situations or alternatives. In general, a measure deemed suitable for one question may be wholly unsuited for another, so there is no general solution to the problem of developing suitable measures of combat effectiveness. A large number of measures relating to battle processes such as engagements, movements, and communications, are in use today. Many of these measures are defined in terms of a single average or accumulation over an entire battle, such as "average number of rounds fired per engagement," or "casualty exchange ratio." Even measures involving more than a single summary value, such as "distribution of ranges at engagement," may not provide a sense of the evolution of the battle over time.

The analyst's view seems to contrast with the historian's view, in which the development of battles and relative movements and contacts of forces are emphasized. These are sometimes summarized on maps of the battlefield using arrows and other symbols to represent major events such as force movements or attacks. Yet, many measures of effectiveness used by military analysts to quantify the performance of military forces in combat are, at their roots, derived from historical context. This stems from the efforts of historians to account for the performance of combatants seen through history; so many of each type at the start of battle, so many killed at its conclusion. Such summary analyses of battles are the beginnings of explanatory models developed to provide understanding of why some battles result in victory while others mire in defeat.

Both historical inspections and analytical models may fail to capture many factors such as leadership, will, fear and suppression, which are relevant (or even of paramount importance) to battle dynamics and ultimate battle outcomes. This is reflected in our inability to explain much of the variation in battle measures and our insistence on using several different measures in evaluations related to combat. Certainly, useful insights may be afforded by even very simple models. For example, Hatzopoulos [ref. 6] develops a modern naval combat model which incorporates some aspects of human factors and shows they can definitely influence which side wins the battle. Another example is the Lanchester family of models which we describe next as background for our development of a proposed measure called the "battle trace."

Lanchester models are analytical models of battles in which the casualty process is envisioned as a continuous erosion of force levels on each side due to fires from the opposing side [ref. 7]. The changes in force levels caused by the casualty processes are modeled in terms of systems of differential equations with an independent time variable and various terms and parameters relating to force sizes, effectiveness of fire and logistical considerations. (A good review of applications of Lanchester theory with mathematical emphasis on the validity of assumptions underlying the models is given by Ancker and Gafarian [ref. 1]. See also Taylor [ref. 9] for a rather exhaustive treatise on Lanchester theory.) Even though such Lanchester models are simplistic, they are sometimes used as building blocks in more elaborate models [ref. 8]. They can also be used to gain important insights into general combat considerations [ref. 9], and they have been found to provide good fit to force size changes actually observed in certain battles [ref. 4].

It must be remembered that the information available to historians is, at best, a summary statement of what happened with limited information as to why. Certainly, Lanchester was similarly motivated to build models of explanatory power, albeit with mathematical tools. Both Lanchester and historians rely primarily on the sizes of forces engaged and a count of resulting attrition over time as the battle progresses. It is not surprising, given this evolution of quantitative battle analysis, that the same information is the basis of other analytic measures of combat effectiveness. Notable examples are ratio measures used to define force effectiveness, such as loss exchange ratios and system exchange ratios. These and similar measures are the most common measures of effectiveness employed to assess the potential utility of new weapon systems and force structures.

Exchange ratio measures are popular in part because of their relationship to both the Lanchester and historical contexts, and in part because they are easy to compute and interpret. They are also recognized to be deficient in explanatory power in that they aggregate the results of battle processes in ways which tend to obscure the detail of battle. This makes any analysis of cause and effect difficult. Furthermore, their computation generally relies on post hoc analysis of battle results. This means they are of little use in real-time applications where measures are needed during the course of battle for training or command system applications. These criticisms are not an indictment of past analyses which have used these ratio measures of effectiveness. Indeed, the traditional ratio measures are very useful in analyses of battle results. Rather, it is an admission that a significant amount of information is lost to such ratios and that there is room for measures which track the battle process through time. This is especially important when the trace of the event history of a battle can be captured or reconstructed exactly. This is possible, for example, when high resolution combat simulations are used for the analysis of weapon capability, force structure and tactics.

The problem is to define new measures of effectiveness which overcome some of the aforementioned shortcomings. The battle trace measure we introduce below makes use of greater underlying information content of a battle, yet preserves the intuitive appeal of those measures suggested by both Lanchester and historical analyses. We observe that the more classical measures of effectiveness do not take into consideration much detailed information about the spacial and temporal nature of a battle. Save for knowledge of the aggregate force sizes prior to combat, the resulting attrition produced during the battle, and, perhaps, the duration of the fight, little additional information as to the dynamic interactions between weapon systems at various points during the battle can be discerned from most traditional measures of effectiveness. Short of being a "replay" of the battle, quantitative measures are needed which can illuminate:

who could see whom, when, where, with what weapons;

what shots were fired, when, where, at what ranges; and

who died, when, where, and from what cause?

Beyond mere reporting of the battle as it occurred, better measures of effectiveness might also demonstrate effects of battlefield synergism which may exist between weapons systems and the results of the synchronization of battlefield operating systems. Battlefield operating systems include: intelligence, maneuver, fire support, air defense, mobility/counter mobility/survivability, combat service support, and command and control [ref. 10]. Certainly, it would also be desirable if the measures which contain this information allow for some comparison between different battles and have an appealing referent baseline from which useful comparisons may be made.

In this paper, we use a simple Lanchester model to motivate our development of the "battle trace" measure of combat effectiveness. We then consider measuring differences in battles in terms of a distance between their corresponding battle traces, and apply this concept to battle data obtained from a Lanchester simulation. Finally, we apply these ideas to comparisons of effects in designed experiments, in the relatively simple context of introduction of human factors into the Janus combat simulation model.

2 LANCHESTER SQUARE-LAW

In analyzing any single battle, or any collection of battles, we wish to use a measure of effectiveness appropriate for deciding which side is winning at a given moment, or which side would win overall if the battle continued for a long time with the present characteristics. This measure of effectiveness should quantify in some way the "combat power" of the opposing forces. With an accurate measure of power, the force with the greater power at some point in the battle is "winning" at that given moment. To clarify the discussion let's consider one important analytical model, the basic Lanchester square-law model. Many of the important and widely used analytical or simulation models incorporate features of this Lanchester model, so it has a certain generic quality relevant to the discussion.

The Lanchester square-law model is governed by the first-order system of differential equations given by

$$dx/dt = -ay; \quad a>0, \quad x(0) = x_0,$$
$$dy/dt = -bx; \quad b>0, \quad y(0) = y_0. \tag{1}$$

In the system, x(t) and y(t) denote the strengths of the forces X and Y at time t, respectively. Assume that the strength of a force is simply the number of combatants in operation at time t. The situation is idealized by assuming that x(t) and y(t) can take on real, not just non-negative integer values. That is, we assume x(t) and y(t) are both continuously differentiable functions of t, so they are "smooth" functions without corners or cusps on their graphs. The positive constants a and b are called the **attrition-rate** or **weapon-kill coefficients**. The larger the value of a, for instance, the more effective is the Y force in eliminating its opponent X force. Of course, in reality, the attrition-rate coefficients are not likely to be constant over time and involve human factors as well as weapon technologies. It is also improbable that attrition rates can be measured to a high degree of precision. To find an analytical solution to the model (1), set

$$dy/dx = (dy/dt)/(dx/dt) = -bx/(-ay)\ . \tag{2}$$

Separating the variables in (2) yields

$$ay\ dy = bx\ dx. \tag{3}$$

Integrating this last equation results in

$$ay^2 - bx^2 = c, \tag{4}$$

and substitution of the initial force strengths x(0) and y(0) provides the value of the constant of integration:

$$c = ay_0^2 - bx_0^2. \tag{5}$$

Typical solution trajectories (x(t),y(t)) in the phase plane are depicted in Figure 1.

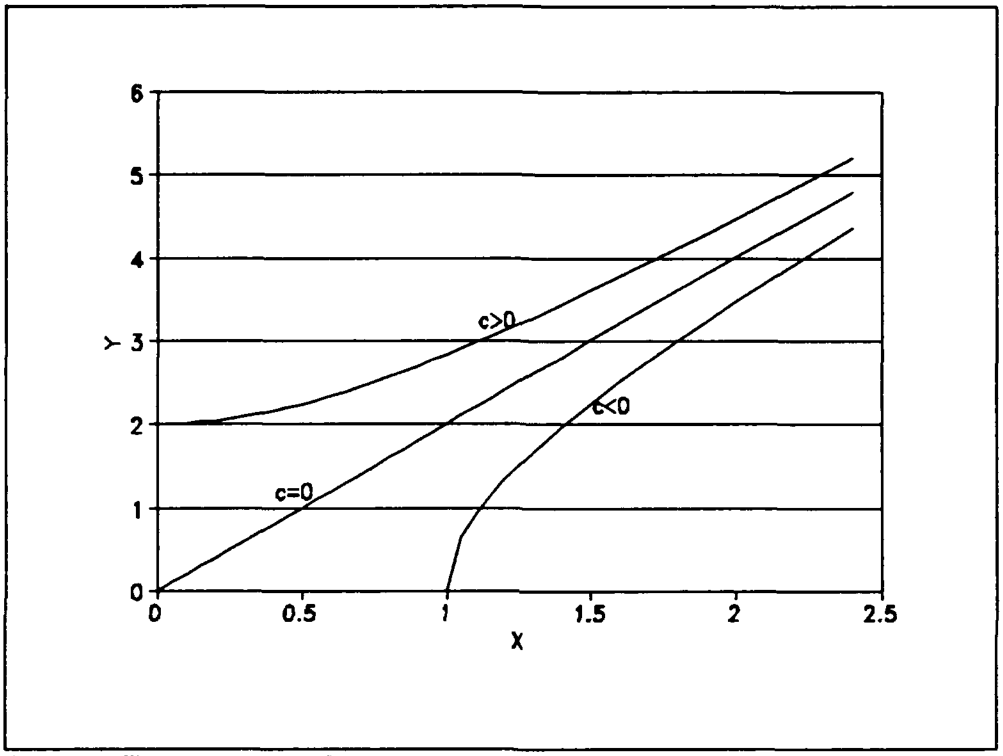

Figure 1. Trajectories of the Lanchester square-law model are hyperbolas in the phase plane when $c \neq 0$.

The trajectories for $c \neq 0$ are hyperbolas, and when $c=0$, the trajectory is the line $y = \sqrt{b/a}\,x$. When $c>0$, the trajectory intersects the y-axis at $y = \sqrt{c/a}$; then the Y force wins with the X force being annihilated. On the other hand, if $c<0$, the X force wins with a final strength level of $x = \sqrt{c/b}$.

Now consider the case where the Y force wins. The constant c must be positive, so

$$(y_0/x_0)^2 > b/a\,. \tag{6}$$

The inequality (6) gives a necessary and sufficient condition that Y wins, under the assumptions of the model (so that reinforcements are not permitted, for example). From the inequality one can see that a doubling of the initial Y force level, or y_0, results in a four-fold advantage for that force, assuming the X force retains its same initial level, x_0. This means that the X force must increase its attrition-rate coefficient b (that is, its technology, training, and overall effectiveness) by a factor of 4 in order to keep pace with the increase in the size of the Y force, assuming the initial strength x_0 is kept at the same level. Figure 2 depicts a typical graph showing the force levels x(t) and y(t) versus time when inequality (6) is satisfied. Observe in the figure that it is not necessary for the initial Y force level y_0 to exceed the level x_0 of the X force in order to ensure victory for Y. The crucial relationship is that the initial force levels satisfy inequality (6).

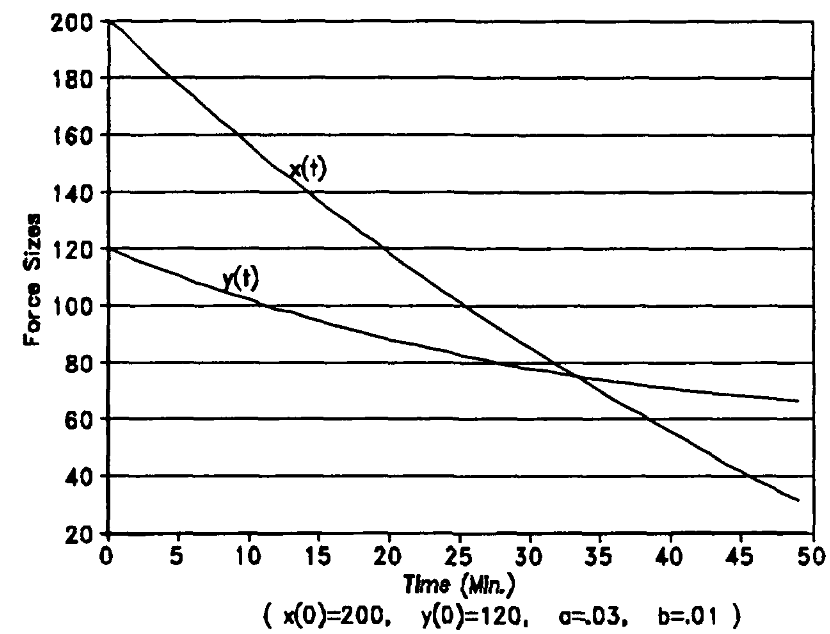

Figure 2. Force levels versus time, for c > 0.

The model (1) can be converted to a single second-order differential equation in y as follows: differentiate dy/dt in (1) to obtain

$$d^2y/dt^2 = -b(dx/dt) = -b(-ay)$$

or

$$d^2y/dt^2 - aby = 0.$$

It is easy to solve this last equation using standard methods from elementary differential equations:

$$y(t) = y_0 \cosh \sqrt{ab}\, t \;-\; x_0 \sqrt{a/b} \sinh \sqrt{ab}\, t \,.$$

Thus,

$$y(t)/y_0 = \cosh \sqrt{ab}\, t \;-\; (x_0/y_0)\sqrt{a/b} \sinh \sqrt{ab}\, t \,. \tag{7}$$

(The solution for $x(t)$ is easily found by interchanging x and y (and a and b) throughout equation (7).) Equation (7) implies that Y's normalized force level, $y(t)/y_0$, depends on an engagement parameter

$$E = (x_0/y_0)\sqrt{a/b}\ , \tag{8}$$

and a time parameter

$$T = \sqrt{ab}\ t\ . \tag{9}$$

The constant $\sqrt{ab}$ represents the intensity of the battle and controls how quickly it is driven to conclusion: the larger $\sqrt{ab}$, the shorter the length of the battle. The ratio a/b represents the relative effectiveness of individual combatants on the two opposing sides.

Inequality (6) suggests considering the more general inequality,

$$(y/x)^2 > b/a\ . \tag{10}$$

If we think of the battle as occurring at discrete time steps, then at each step t we could compare the fraction $(y(t)/x(t))^2$ to the quotient b/a. Alternatively, we rewrite (10) in the form

$$(x/y)^2\ (b/a) = c\ .$$

If $c < 1$, then inequality (10) holds and, at that instant in the battle, the Y force is winning. This observation suggests comparing bx^2/ay^2 against 1. If $bx^2/ay^2 < 1$, then Y is winning at time t; if $bx^2/ay^2 > 1$, then the X force is winning at that point in the battle. When $bx^2/ay^2 = 1$ we have a draw, in the sense that the sides are mutually annihilating one another.

Let us examine further the ratio bx^2/ay^2, since we do not generally know the values of a and b, at least to any high degree of precision. Now,

$$\begin{aligned} bx^2/ay^2 &= [-bx/(-ay)] \cdot x/y \\ &= [(dy/dt)/(dx/dt)](x/y) \\ &\approx [(\Delta y/\Delta t)/(\Delta x/\Delta t)](x/y) \\ &= (\Delta y/\Delta x)(x/y) \\ &= \frac{\dfrac{\Delta y}{y}}{\dfrac{\Delta x}{x}} \end{aligned}$$

where Δy and Δx are Y's and X's losses, respectively, in a time interval of duration Δt.

This last result suggests that we compare the ratio

$$R = \frac{\frac{\Delta y}{y}}{\frac{\Delta x}{x}} \tag{11}$$

against the number 1. We have the following results based on the Lanchester square-law model:

(1) if R < 1, the Y force is winning at time t;

(2) if R > 1, the x force is winning at time t;

(3) if R = 1, the battle is one of mutual annihilation

It is assumed that y and x are positive for any time t at which R is evaluated. If y is zero throughout the time period $[t, t+\Delta t)$, then, in accordance with eq. (1), Δx should also be zero for that particular time interval, and the ratio (11) is not defined. For such cases, various options could be followed: we might define R = 1 for that interval; we might simply suspend computation of R for that interval; or we might let Δt get large enough so both y and Δx are positive. (If y is positive but Δx is zero, then other alternatives might be followed; we will give further consideration to this later.)

Let us interpret these results. If R<1, then $\Delta y/y < \Delta x/x$, so the Y force is suffering less proportional attrition than the opposing X force; that is, Y is winning. If R>1, the situation is reversed and Y is suffering greater proportional attrition compared to the X force; Y is losing. Notice from eq.(11) that if y is very large compared to x, and assuming the total kill capabilities for both sides are about the same so $\Delta y \approx \Delta x$, then R<1 is highly likely. We conclude that "bigger is better" for the Y force, especially when the forces are evenly matched in terms of kill capabilities. This is a reasonable result.

3 BATTLE TRACE

Our preceding analysis suggests we monitor t' e ratio R over the course of a battle. To begin, consider a battle of duration 1 hour, and compute the ratio R every Δt = 5 minutes based on both friendly and enemy attrition. If it happens that Δy and Δx are both zero at the end of a 5-minute time period, we might agree to set R=1 (the stalemate value). If $\Delta x = 0$ but $\Delta y \neq 0$, we might agree to set R=2, and the X force is winning. If either force is completely annihilated (so y=0 or x=0), the other side automatically scores a win and no value of R is computed at that time. The plot of R versus t (time) over the course of a given battle, subject to the above conditions, is the battle trace. This measure is similar to elasticity measures commonly used in economics. Bitters [ref. 3] used this measure, constrained to battle "stages," in connection with his study of the ability of smaller forces to defeat a larger foe by applying the entire weight of its numbers to numerically smaller subsets of the larger force in a sequence of stages. Figure 3 depicts a battle trace for a hypothetical battle and shows regions where X is winning and where Y is winning. Note that use of equation (11) does not require that the attrition coefficients a and b be constant. In actual battles these values vary in response to changing conditions and battle events. But even with attrition <u>functions</u> a(t) and b(t) in place of a and b, the battle trace given in equation (11) remains valid. In several examples described below, we approximate a(t) and b(t) with step functions that can have different values in the various time intervals making up the battle period.

The next several examples present battle traces for 1-hour battles simulated with a sequence of Lanchester square-law sub-battles with parameters changing each 5 minutes. Sets of values of a and b are generated as outcomes on independent normal random variables at $t = 0, 5, 10, \ldots 55$ minutes, where the means of the normal distributions were selected so as give attrition coefficients representing several types of weapons systems on each side. The values selected gave Y a slight advantage. The variances were selected to give considerable "noise" to the

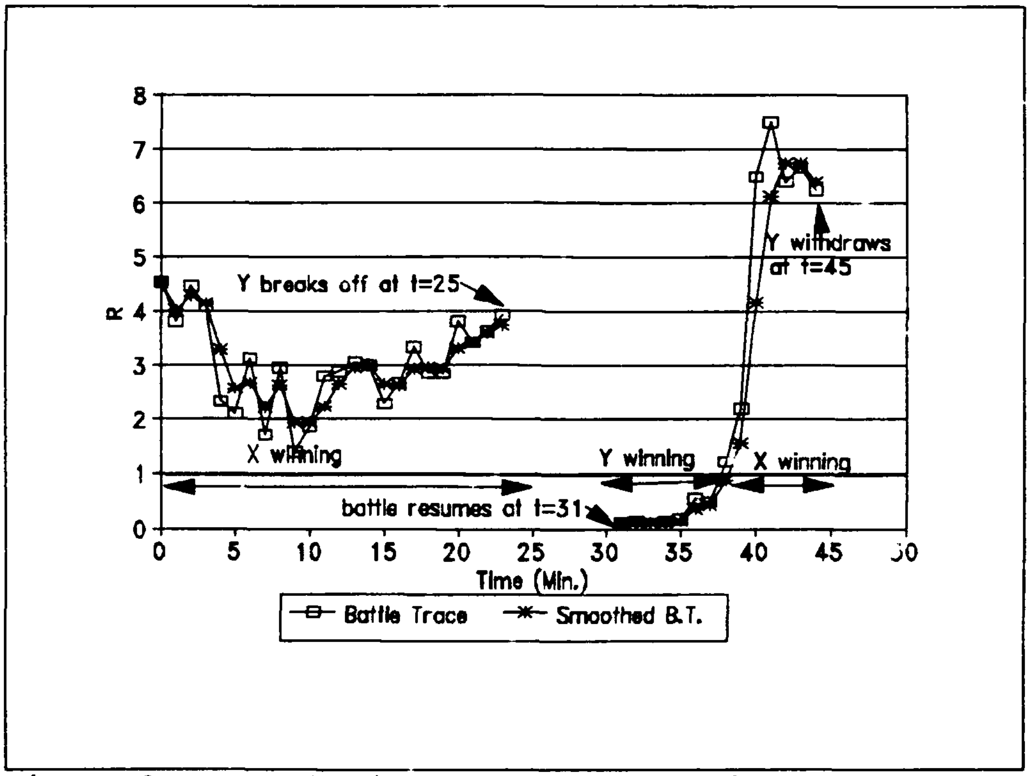

Figure 3. Hypothetical battle trace plot.

casualty process during the course of the battle. During each five-minute period, the force sizes were decremented so as to approximate casualties in accordance with equations (1). That is, in the k^{th} time period, for each weapon system possessed by Y, X was decremented by $\Delta x = -a(t_{k-1})y(t_{k-1})\Delta t$, and similarly for Y. Since a mix of forces was present, each with its own attrition coefficients, the overall force strength for a given side was represented by a weighted sum of strengths of individual weapon systems the side possessed at each given time point in the battle.

These data were generated by Green in connection with other research [ref. 5]. Plots of battle traces for six such simulated battles are shown in Figure 4.

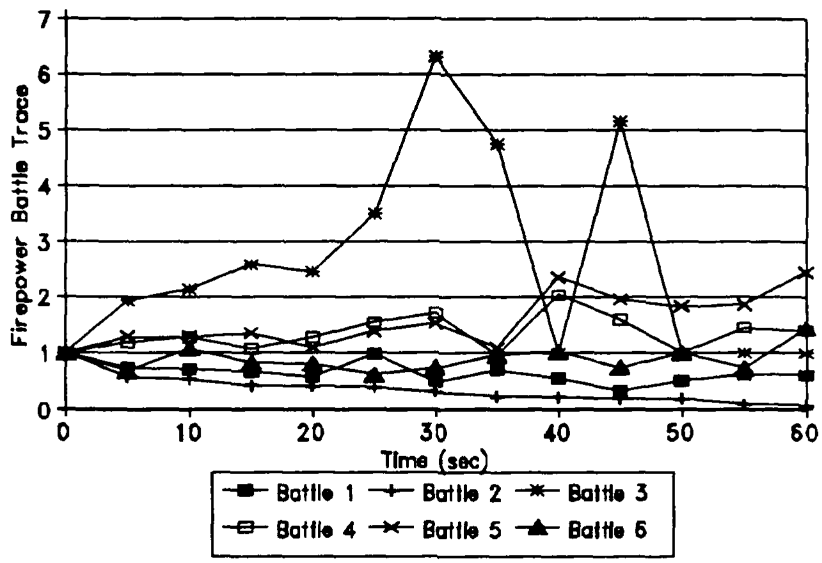

Figure 4. Plots of battle traces for six simulated battles.

It is doubtful that data from real battles are available which would allow computation of this battle trace. However, with simulated battles (in field tests, with combat simulations, and possibly with some training exercises such as those at the National Training Center), such data may be captured. An entire battle may be decomposed into a discrete series of smaller segments (or "mini-battles", [ref. 2]) ordered in terms of times of their occurrences. The time partitions defining the battle segments are somewhat arbitrary. For data from battle simulations, force sizes within each segment need not depend on how many units of each side are present, but only on how many "see" each other. Thus the number of forces x_k and y_k used to calculate the value R_k of the battle trace for the k^{th} time interval (i.e., the k^{th} battle segment) depends on how the forces contact one another on the terrain of the battlefield, not simply on the absolute numbers of forces present. This inter-visibility is a function of the movement, tactics, terrain and synchronization of battlefield operating systems. Computationally, all that is required is a reasonable selection of the time partition size (Δt) and a suitable algorithm for determining the values of x_k and y_k, and possibly Δx_k and Δy_k, in the k^{th} mini-battle.

To obtain values for x_k and y_k, consider the implications of selecting quantities which correspond to the maximum number of respective unique combat systems which have acquired at least one system of the opposing side. This is equivalent to saying that y_k equals the number of effective combatants in the k^{th} battle segment for side Y; that is, y_k is the maximum number of Y's units which have acquired at least one of X's units (or maintained acquisition from the preceding segment). A symmetric definition holds for x_k. To illustrate these ideas, consider the hypothetical situation represented in Figure 5, computed with data in Table 1. Here, at time 1, 13 X combat systems confront the 11 systems of an opponent, Y. In the instant t=1 of the battle depicted, two X systems can "see" at least one Y system, thus $x_1 = 2$. Four Y systems, however, can observe at least one X system, so $y_1 = 4$. We suppose one of X's force and two of Y's force has been killed by the end of the first time interval, Δt_1.

Table 1. Hypothetical battle data.

Time	x(t)	y(t)	Δx	Δy	R
1	2	4	1	2	1
2	2	3	1	2	1.333333
3	4	3	1	2	2.666667
4	5	2	1	1	2.5
5	2	4	3	2	0.333333
6	2	2	2	1	0.5
7	1	1	1	1	1
		Totals	10	11	

During the second period, two X units see Y targets and three Y weapons see X targets; another X system is killed and two more Y systems are casualties. This process continues through seven stages of battle, at which time Y is annihilated.

4 LOG TRACE AND SYMMETRIC BATTLE TRACE

Since decision makers are frequently more familiar with additive symmetry than with reciprocal symmetry, it may be useful to transform the battle trace R to its logarithm, log(R), which we call "log trace." The log trace measure has a simple symmetry between the points of view of the X force commander and the Y force commander. The battle trace is not symmetric about the "mutual annihilation value" of 1, so plots of the battle trace for a given battle viewed from the X and Y points of view are not simply reflections across the line R=1. This means that portions of battle traces falling between 0 and 1 must be interpreted differently than those portions above 1. But plots of log trace have symmetric interpretations viewed from X's and Y's points of view, in an additive sense. The log trace measure indicates a draw at value 0; for traces above 0 the X force is winning, and for negative values of log(R), Y is winning. Other alternatives to achieve symmetry of interpretation are possible. One attractive alternative is to plot R when $R \geq 1$ and $2-R^{-1}$ when $R<1$, for example. This is the "symmetric battle trace" plotted with "+" symbols in Figure 5 and discussed further below. In the remainder of this section, we make further comments on the log trace.

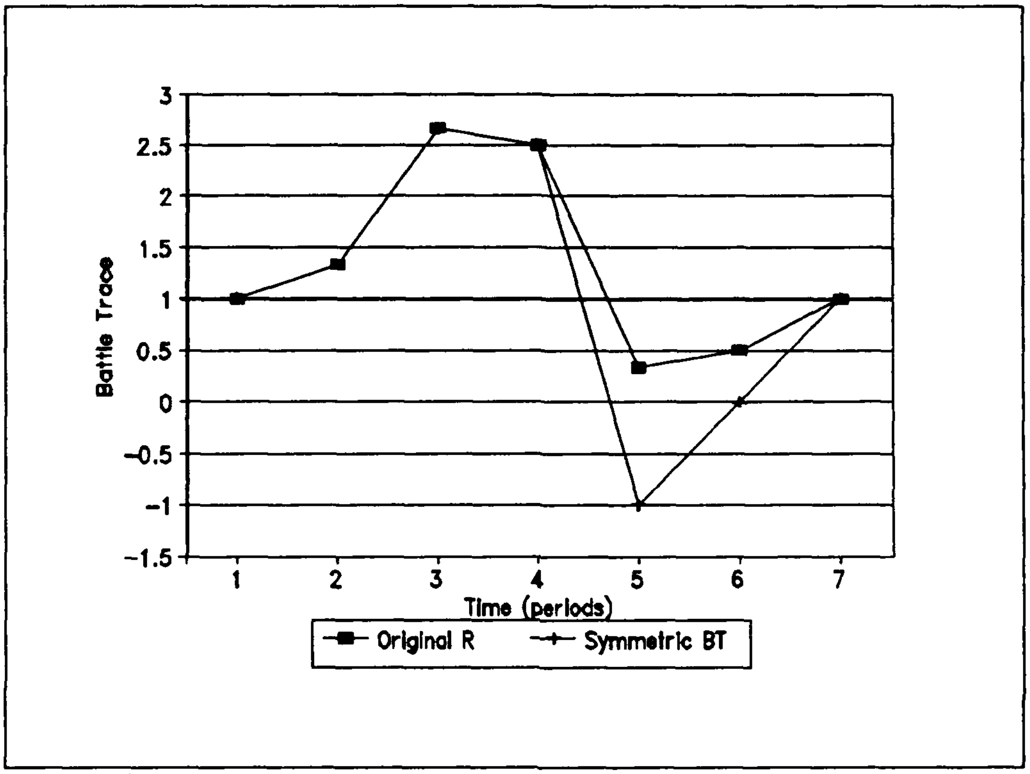

Figure 5. Plot of battle trace (square symbols) corresponding to data shown in Table 1.

Since the domain of the logarithm function does not include zero, one must ensure the arguments involved are not zero at any point in the battle for which log trace is to be computed. That is, noting, for the k^{th} segment,

$$\log(R_k) = \log(\Delta y_k) - \log(y_k) - \log(\Delta x_k) + \log(x_k),$$

the possibility that any of the terms Δy_k, y_k, Δx_k or x_k are zero must be avoided. This can be accomplished in any of several ways. One might control the time intervals Δt so they are sufficiently large that none of the arguments is zero. This could include taking the Δt's to be of differing widths, possibly depending on the data involved, if needed. One might simply suspend calculation of log trace for time intervals in which one or more of the arguments is zero. Yet another possibility is to add a small positive quantity to each of these terms before taking their logarithms. For example, consider adding 1.0 to each term, so when a given term such as Δy_k is zero, the contribution to log trace, $\log(\Delta y_k + 1) = \log(1)$, is zero. Thus, with this definition, the log trace measure of battle effectiveness is, within the k^{th} battle segment, log trace_k =

$$\log(\Delta y_k+1) - \log(y_k+1) - \log(\Delta x_k+1) + \log(x_k+1). \qquad (13)$$

Below, we consider how statistical inferences about battles, and in particular about differences between battles, may be made, using the battle trace or log trace measures. It is anticipated that the log trace measure may have simpler statistical characteristics than would have the battle trace measure, because sums of random quantities tend to have relatively more simple distributions than have quotients of random quantities. Indeed, in many applications it may be found that the log trace measure is normally distributed within a given battle segment, at least to a rough approximation. This may be due in part to the central limit theorem, and in part to the individual random variables whose logarithms are taken in equation (13) being approximately log-normally distributed. We also consider nonparametric comparisons of battles using the battle trace, symmetric battle trace and log trace, below.

5 COMPARISONS OF BATTLES

We have discussed the difficulty of the task of measuring battle effectiveness, and have made the point that no single simple measure can capture all that is of interest. This holds for the battle trace measures as well, of course. Nevertheless, we believe comparisons of battles, in terms of their battle traces or log traces, can contribute to an understanding of the nature of the differences in the battles. In what follows, we discuss the use of symmetric battle traces, defined by

$$R_s = \begin{cases} R, & if\ R > 1; \\ 2 - R^{-1}, & if\ R \quad 0. \end{cases}$$

The principles below can also be applied to battle traces and log traces. We will refer to R_s as the "battle trace" in the discussion below.

We envision a context in which a designed experiment has been conducted to examine the effects of introducing human factors into a combat simulation. Specifically, we consider an application of these ideas to the incorporation of a new module into the Janus battle simulation model; this module is designed to play fratricide (the killing of one's own forces), as described in a separate section below.

How might one compare two battle traces, one from a battle fought under condition A (say, Janus with the new fratricide module), and one battle fought under condition B (Janus without fratricide)? One possibility is to define a notion of the distance between their respective battle traces. In order to make sensible comparisons, it may be necessary to transform the time domains of the battles under comparison so that they have comparable battle start and battle end times. For simplicity, we assume this transformation has been done, and agree to choose a time scale which makes t=0 the battle start time and t=1 the battle end time. More exotic transformations of the time scales may be necessary to "synchronize" the battles under comparison, but we do not consider this further here. For the comparisons of Janus battles considered in this paper, we have set the simulation parameters so that matching the battle start times and stop times with the points 0 and 1, respectively, in all battles under consideration, appears to give adequate synchronization of the battles.

A set of possible distance functions can be based on the L_p metrics used in mathematical analysis. With this approach, the distance between two battle traces, say $bt_A(t)$ and $bt_B(t)$ is given by

$$\|bt_A - bt_B\| = \left[\int_0^1 |bt_A(t) - bt_B(t)|^p \, dt\right]^{1/p} \tag{14}$$

We suggest that the L_1, L_2 or L_∞ distance functions may be good candidates for use in comparing battles. The L_∞ distance function, obtained by letting $p \rightarrow \infty$, gives the distance between two battle traces as the supremum (over $t \in [0,1]$) of the vertical distance between the traces: $\sup_{t\in[0,1]} | bt_A(t) - bt_B(t) |$. An interesting alternative notion of distance, involving the L_∞ distance between fitted probability surfaces related to battle outcomes, has been studied by Green [ref. 5].

The application discussed below uses the L_1 distance function, which is just the (absolute) area between the two battle traces. This definition of distance has the advantage of being interpreted easily in terms of the graphs of the battle traces for the two individual battles. We also consider a "score distance," defined in terms of the difference between scores computed for each battle trace, as follows. Consider whether the battle trace is above 1 (X winning) or below 1 (X losing) and whether the battle trace is increasing (X improving relative to Y) or decreasing (X worsening relative to Y). Then measure the number of time increments the battle trace "occupies" each of the four combinations of these conditions (see Table 2) multiplied by the weights (assigned arbitrarily) in the four cells shown in Table 2. The **score** for a given battle trace is the sum of the occupancy times in each cell multiplied by the weight for that cell.

As an example, consider the data shown in Table 1, using the symmetric battle trace measure and counting whether the trace is

Table 2. Battle trace score weights.

log trace	positive	negative
increasing	3	-1
decreasing	1	-3

above or below 1, rather than positive or negative. It can be seen that the trace is above 1 and increasing during two periods (prior to t=2 and prior to t=3); it is above 1 and decreasing in one period (prior to t=4); the trace occupies the upper right-hand corner two periods and the lower right cell one period. Therefore, with the weights shown in Table 2, the score for this battle is

2(3)+1(1)+2(-1)+1(-3) = 2, suggesting X enjoyed relatively better success in the battle than did Y.

6 DETERMINING SIGNIFICANCE IN DESIGNED EXPERIMENTS

The distance between battle traces cannot be interpreted in operational terms until the distribution of distances is determined for battles from the same source (i.e., where the null hypothesis of no difference in battles is true). For our application to comparisons of battle traces from two versions of Janus, we need to establish the distributions of distances within- and between-treatments, where "treatments" refer to the presence or absence of the fratricide module. One possibility is to generate a large sample of independent battles, using Janus, for each treatment. Then the distributions of distances or scores for each treatment can be compared. This comparison might be accomplished using the Kolmogorov-Smirnov test of the hypothesis of no difference in distributions of distances. Other non-parametric tests such as the Mann-Whitney test or Kruskal-Wallis analysis of variance by ranks could also be applied. If the assumptions of normality and homogeneity of variance appear to be tenable, a standard test such as the t-test might be useful.

We believe a randomization test may be a good choice for comparisons of battle trace distances. A randomization test works as follows: Suppose we have a random sample of n observations from population 1 and a random sample of m observations from population 2, and suppose we wish to test the hypothesis that the populations are the same. Suppose further that we have defined a test statistic that we believe should tend to be large (positive) when the populations are different, relative to values obtained when the hypothesis is true. Let the computed value of the test statistic be t_0, using the data from the two observed samples. Now pool the data in the two samples, and draw a random sample of size n (without replacement) from the pooled data. This will determine two "pseudo samples" of sizes n and m. Compute the test statistic again, as if the two pseudo samples drawn from the shuffled pooled data had been the original samples, and let t_1 be the resulting "pseudo" value. Now draw a second pseudo sample of size n at random (without replacement) from the pooled data, and compute the corresponding test statistic pseudo value, t_2. Continue in this fashion for some large number N of times. Next, form a histogram of the computed pseudo test statistics t_1, t_2, ..., and determine the position of the actual test statistic value, t_0, in this histogram. If the actual value is in the upper tail of the sample of pseudo values, one may conclude the populations are different (that is, the value of the test statistic is extraordinarily large, relative to values obtained when there is no difference in the sources of the two samples of sizes n and m).

A randomization test using the L_1 distance between battle traces for ten samples from each of two Lanchester simulations is discussed in the next section.

7 SIMULATED BATTLE TRACES FOR BATTLES WITH AND WITHOUT FRATRICIDE

A simulation was written to generate battle traces in accordance with Lanchester "mini-battles" within time intervals of width 10^{-2}. The general scenario envisioned involves an attack by 400 Blue units upon a Red defending force consisting of 100 units. In the early part of the battle, Red has higher proportions of acquisition of Blue targets and has higher attrition coefficient (a in eqs. (1)). As the battle progresses, the superior numbers of the attacking force begin to take a toll upon the numbers of surviving Red defenders. As Blue closes with Red, the Red force begins to withdraw and the attrition coefficient (b) of Blue increases while that of Red decreases. At the beginning of the battle, the Red attrition coefficient is .05 and that for Blue is .01. These values are adjusted stochastically as the battle progresses so that near the end of the battle these values are, on average (over many runs of the simulation), approximately equal to .03. Additional stochastic variation in the simulation outputs was induced by drawing, for each period of the battle, the number of Red units that could see at least one Blue unit, y in equation 11, from an appropriate distribution which depended on the number of Red and Blue units alive at the beginning of the period. The values of x in eq. (11) were similarly determined stochastically as functions of the numbers of Blue and Red units still alive, as the battle progressed.

Finally, the simulation was modified to represent fratricide, for half of the simulation runs. This was done by further degrading the size of the Blue force at the end of each battle period so as to decrease it by .05%, representing losses due to fratricide. Over the course of 100 periods of battle, the overall level of fratricide losses to the Blue force amounted to approximately 3% of the initial Blue strength. Plots of the symmetric battle trace for three battles are shown in Figure 6. One symmetric trace is for the simulation without fratricide; one is for a paired battle with fratricide (paired on a random number stream used in the simulation); and one symmetric battle trace is for a battle with fratricide, with an independent random number stream.

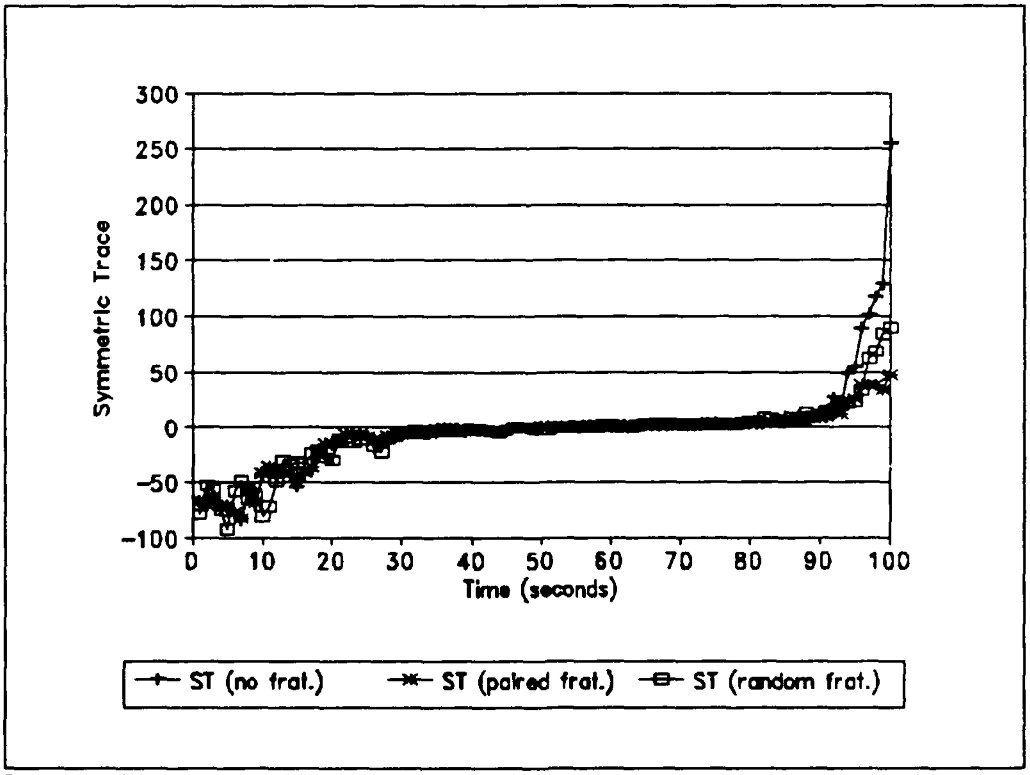

Figure 6. Symmetric battle traces for three simulated battles.

8 COMPARING SIMULATION WITH AND WITHOUT FRATRICIDE

To illustrate how battle traces might be used to determine whether there is a difference between simulation versions, we ran the simulation described above to generate symmetric battle traces for 10 battles without fratricide and 10 battles with fratricide. These battle traces were used to conduct several tests of the null hypothesis that there is no difference between the simulated battles with and without-fratricide. The purpose of these illustrations is to show how the tests suggested in preceding sections might be applied to comparisons of Janus(A) and Janus(A) modified to include fratricide.

We computed the scores for each battle trace in accordance with the weights shown in Table 2; the results are as follows:

> no fratricide: -16, -18, -8, 2, -36, -52, -16, -28, 10, -14;
> fratricide: -42, -36, -10, -32, 8, -52, -24, -20, -24, -2.

These scores were compared using a Mann-Whitney test. The sum of ranks for the ten scores corresponding to simulations with fratricide is 92, which is significant at the .16 level (one sided). Thus the scores test does not indicate there is a significant degradation in scores due to fratricide, as played in the simulation.

Two alternate tests were based on the areas between symmetric battle traces; that is, with the L_1 distance between the symmetric battle traces. In the first alternate test, a set of 10 distances was computed between pairs of randomly selected traces, one from the set of traces for battles without fratricide, and one from the set of traces for battles with fratricide. A second set of areas was constructed using pairs of traces from battles without fratricide and pairs of traces for battles with fratricide. A two-sample t-test was conducted with the summary statistics as follows:

> mean of 10 areas for fratricide-non fratricide pairs: 686.25
> (standard deviation = 163.07)
>
> mean of 9 areas for fratricide pairs: 615.6
> (std. dev. = 117.37)
>
> mean of 9 areas for non-fratricide pairs: 635.4
> (std. dev. = 92.16)
>
> $t = (686.25 - 625.50)/73.35 = .83$; d.f.=25.

This value of t corresponds to a significance level of .22 (one sided), which agrees with the conclusion of the scores test above.

A second test, based on the L_1 distance between fratricide and non-fratricide symmetric battle traces, was conducted using the randomization test described above. For this test, the test

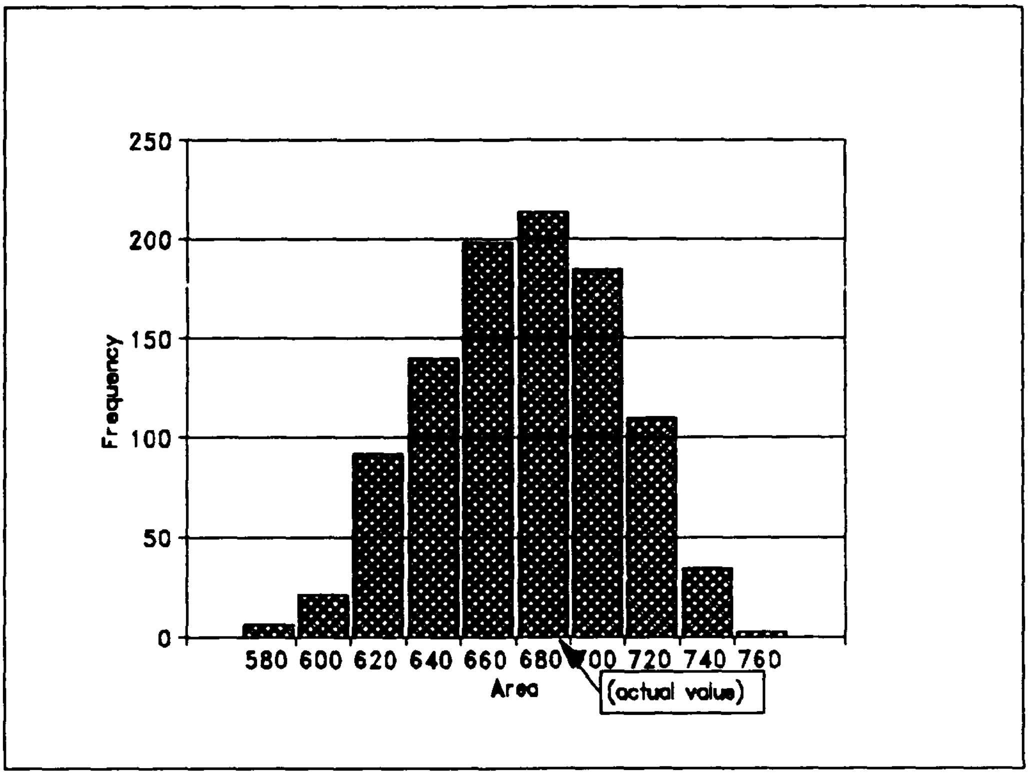

Figure 7. Histogram of pseudo test statistic values.

statistic was taken to be the average of 10 (absolute) areas between symmetric battle traces. The actual test statistic, t_0, had value 686.27. An additional 1000 values of the test statistic were calculated. For each pseudo value, t_1, t_2, ..., ten pairs of symmetric battle traces were selected at random without replacement from the pooled collection of 20 battle traces and the average of the L_1 distances was computed. The histogram in Figure 7 shows the distribution of pseudo values of the test statistic. The actual value, t_0 = 686.27 falls at the 1 -.367 quantile of this distribution, so the randomization test gives significance level .37 for the test (two-sided). Again, this is consistent with the conclusion made with the scores test.

9 CONCLUSION

The battle trace appears to have potential for use in modeling and evaluation applications. It is interesting to note that this application of the Lanchester combat model does not require determination or estimation, before hand, of the attrition coefficients which are typically quite difficult to obtain and justify. The notion that the attrition coefficients are not static, but rather change over time as the battle progresses is important. Useful analytical models of combat might arise through considerations of how these coefficients change in response to battle events.

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